

# A new approach to calculate the gluon polarization

---

**F. Taghavi-Shahri<sup>a</sup>, A. Mirjalili<sup>a,b</sup> and M. M. Yazdanpanah<sup>a,c</sup>**

<sup>a</sup> *School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM) P.O. Box 19395-5531, Tehran, Iran*

<sup>b</sup> *Physics Department, Yazd University, Yazd, Iran*

<sup>c</sup> *Physics Department, Kerman Shahid Bahonar University, Kerman, Iran*

*E-mail: f\_taghavi@ipm.ir, Mirjalili@Mail.ipm.ir, myazdan@mail.ipm.ir*

**ABSTRACT:** We derive the Leading-Order master equation to extract the polarized gluon distribution  $G(x, Q^2) = x\delta g(x, Q^2)$  from polarized proton structure function,  $g_1^p(x, Q^2)$ . By using a Laplace-transform technique, we solve the master equation and derive the polarized gluon distribution inside the proton. The test of accuracy which are based on our calculations with two different method confirms that we achieve to the correct solution for the polarized gluon distribution. We show that accurate experimental knowledge of  $g_1^p(x, Q^2)$  in a region of Bjorken  $x$  and  $Q^2$ , is all that is needed to determine the polarized gluon distribution in that region. Therefore, to determine the gluon polarization  $\frac{\delta g}{g}$ , we only need to have accurate experimental data on un-polarized and polarized structure functions ( $F_2^p(x, Q^2)$  and  $g_1^p(x, Q^2)$ ).

**KEYWORDS:** Parton Model, Polarized gluon distribution, Laplace transform.

---

## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. LO master equation to extract the polarized gluon distribution <math>G(x, Q^2) = x\delta g(x, Q^2)</math>, using the polarized proton structure function, <math>g_1^p(x, Q^2)</math></b>	<b>2</b>
<b>3. Solution of the master equation</b>	<b>3</b>
<b>4. Numerical results</b>	<b>5</b>
4.1 Global parametrization of $g_1^p(x, Q^2)$ using all available experimental data	5
4.2 Numerical results for the polarized gluon distribution	6
4.3 How to test the accuracy of the procedure ?	7
<b>5. Conclusion</b>	<b>9</b>
<b>A. The DGLAP equation for the polarized proton structure function</b>	<b>10</b>

---

## 1. Introduction

The spin structure of the proton is one of the most challenging open puzzles in Quantum Chromodynamics. One of the main questions in particle and nuclear physics is: How is the proton spin built up from its quark and gluon constituents? Now, we know that the quark contribution to the spin of the proton is about 0.30. It is determined precisely in a QCD fit to the polarized proton structure function data,  $g_1^p(x, Q^2)$ . Therefore at the present time, the role played by the gluons in the nucleon spin is the most challenging task.

The main goal of the experiments at HERMES, SMC, COMPASS and RHIC spin physics programs is the measurement of the helicity contribution of the gluons to the nucleon spin,  $\Delta g$ . Experimentally this value is mainly accessible via two processes in polarized DIS experiments. The first one is the production of high  $p_t$  hadron pairs with large transverse momentum, but these processes have large background contributions from QCD Compton processes and fragmentation that one has to control. The second one is open charmed meson production in photon-gluon fusion process. The cross section of these two processes is directly related to the ratio of the polarized gluon density to the un-polarized gluon density,  $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$  [1, 2]. Then, to measure the gluon contribution to the nucleon spin with this approach, it is necessary to know the un-polarized gluon distribution well. Polarized gluon measurements from deep inelastic experiments are summarized in Table 1.

In line with experiments, many NLO QCD global fits to the inclusive  $g_1^p(x, Q^2)$  data were used to extract the magnitude of  $\Delta g$  and the shape of  $\delta g(x, Q^2)$  [8, 9, 10]. Unfortunately,

Experiment	process	$\langle x_g \rangle$	$\langle \mu^2 \rangle$	$\frac{\delta g(x_g, \mu^2)}{g(x_g, \mu^2)}$	Reference
HERMES	hadron pairs	0.17	$\sim 2$	$0.41 \pm 0.18 \pm 0.03$	[3]
HERMES	inclusive hadrons	0.22	1.35	$0.071 \pm 0.034 \pm 0.105$	[2]
SMC	hadron pairs	0.07	—	$-0.20 \pm 0.28 \pm 0.10$	[4]
COMPASS	hadron pairs, $Q^2 < 1$	0.085	$\sim 3$	$0.016 \pm 0.058 \pm 0.055$	[5, 6]
COMPASS	hadron pairs, $Q^2 > 1$	0.13	—	$0.06 \pm 0.31 \pm 0.06$	[5, 6]
COMPASS	open charm	0.15	13	$-0.57 \pm 0.41 \pm 0.17$	[7]

**Table 1:** Polarized gluon measurements from deep inelastic experiments.

these global fits are sensitive to the initial assumption for the polarized gluon distribution and also to the initial scale of  $Q_0^2$ . The shape of the polarized gluon distribution extracted from different groups are not identical because of their different assumptions for the initial polarized gluon distribution, initial  $Q_0^2$  and also different approaches for global fits. We would like to know how big the gluon spin contribution is to the total spin of the proton and look for a simple way to measure this value. This work can be considered as a proposal for direct measurement of the polarized gluon distribution inside the proton with more accuracy.

Recently an explicit expression for the un-polarized gluon distribution function  $G(x, Q^2) = xg(x, Q^2)$  in the proton in terms of the proton structure function  $F_2^p(x, Q^2)$  was derived by using a Laplace-transform technique [11, 12, 13]. Here, the same procedure is used to derive the polarized gluon distribution function,  $G(x, Q^2) = x\delta g(x, Q^2)$ , inside the proton. We obtain an analytic solution for the polarized gluon distribution in terms of the polarized proton structure function,  $g_1^p(x, Q^2)$ . Thus we can calculate the polarized gluon distribution inside the proton directly only by finding  $g_1^p(x, Q^2)$  over large kinematic range of  $x$  and  $Q^2$ . This is the principal theoretical result in this paper.

This paper is organized as follows. In section 2, we derive LO master equation for extracting the polarized gluon distribution  $G(x, Q^2) = x\delta g(x, Q^2)$  from polarized proton structure function,  $g_1^p(x, Q^2)$ . Section 3 is devoted to the solution of the master equation. In the last section, we first perform a global parametrization of  $g_1^p(x, Q^2)$  using all available experimental data and then we calculate numerically the polarized gluon distribution. Our conclusion is given in section 5.

## 2. LO master equation to extract the polarized gluon distribution $G(x, Q^2) = x\delta g(x, Q^2)$ , using the polarized proton structure function, $g_1^p(x, Q^2)$

The LO DGLAP equation, [14, 15, 16] for the evolution of the polarized proton structure function,  $g_1^p(x, Q^2)$  can be written as (See Appendix A)

$$x \frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2) \delta K_{qq} \left( \frac{x}{z} \right) = x \frac{\alpha_s}{2\pi} \sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg} \left( \frac{x}{z} \right), \quad (2.1)$$

where  $\delta K_{qq}(x)$  and  $\delta K_{qg}(x)$  are the LO polarized splitting functions and  $\alpha_s$  is the renormalized running coupling constant. We introduce  $\mathcal{G}_1^p(x, Q^2)$  as

$$\mathcal{G}_1^p(x, Q^2) = x \frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2) \delta K_{qq}\left(\frac{x}{z}\right). \quad (2.2)$$

Now we can write the master equation, the DGLAP equation for the  $g_1^p(x, Q^2)$ , as follows

$$gg(x, Q^2) = x \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg}\left(\frac{x}{z}\right), \quad (2.3)$$

where  $gg(x, Q^2) = (\frac{\alpha_s}{2\pi} \sum e_i^2)^{-1} \mathcal{G}_1^p(x, Q^2)$ . In the Eq. (2.2) and Eq. (2.3), the LO  $q \rightarrow q$  and  $g \rightarrow q$  polarized splitting function are given by [17]

$$\delta K_{qq}(x) = \frac{4}{3} \left( \left( \frac{1+x^2}{1-x} \right)_+ + \frac{3}{2} \delta(x-1) \right), \quad (2.4)$$

$$\delta K_{qg}(x) = \frac{1}{2} (2x-1) = x - \frac{1}{2}. \quad (2.5)$$

To extract the polarized gluon distribution,  $G(x, Q^2) = x \delta g(x, Q^2)$  from the DGLAP equation, we should solve the master equation (2.3) and find the polarized gluon distribution inside the proton. This issue is the subject of the next section.

### 3. Solution of the master equation

To solve the master equation (2.3), we follow the procedure that was used by BDM [11] and used the Laplace transformation to solve this equation. Now we use the coordinate transformation as

$$v \equiv \ln(1/x), \quad (3.1)$$

Then the functions  $\hat{G}$ ,  $\hat{K}_{qg}$ , and  $\hat{\mathcal{G}}$  in  $v$ -space are given by

$$\hat{G}(v, Q^2) \equiv G(e^{-v}, Q^2), \quad (3.2)$$

$$\delta \hat{K}_{qg}(v, Q^2) \equiv \delta K_{qg}(e^{-v}, Q^2), \quad (3.3)$$

$$\hat{\mathcal{G}}(v, Q^2) \equiv gg(e^{-v}, Q^2). \quad (3.4)$$

Explicitly from Eq. (2.5), we have

$$\delta \hat{K}_{qg}(v, Q^2) = e^{-v} - \frac{1}{2}. \quad (3.5)$$

Therefore

$$\hat{\mathcal{G}}(v, Q^2) = \int_0^v \hat{G}(w, Q^2) e^{-(v-w)} \delta \hat{K}_{qg}(v-w) dw = \int_0^v \hat{G}(w, Q^2) \hat{H}(v-w) dw, \quad (3.6)$$

where  $w \equiv \ln(1/z)$  and  $\hat{H}(v)$  is defined as

$$\hat{H}(v) \equiv e^{-v} \delta \hat{K}_{gg}(v) = e^{-2v} - \frac{1}{2} e^{-v}. \quad (3.7)$$

If we take the laplace transform of Eq. (3.6), then we have

$$\mathcal{L}[\hat{\mathcal{G}}(v, Q^2); s] = \mathcal{L}[\int_0^v \hat{G}(w, Q^2) \hat{H}(v-w) dw; s] \quad (3.8)$$

$$\implies \hat{\mathcal{G}}(s, Q^2) = \hat{G}(s, Q^2) \times h(s). \quad (3.9)$$

In the above equation we used the following property for Laplace transformation

$$\mathcal{L}^{-1}[F(s)G(s)] = \int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t f(\tau)g(t-\tau)d\tau. \quad (3.10)$$

So we have the polarized gluon distribution in s-space as

$$\hat{G}(s, Q^2) = \frac{\hat{\mathcal{G}}(s, Q^2)}{h(s)}. \quad (3.11)$$

Now the polarized gluon distribution in v-space is given by

$$\hat{G}(v, Q^2) = \mathcal{L}^{-1}[\hat{\mathcal{G}}(s, Q^2)h(s)^{-1}] = \int_0^v \hat{\mathcal{G}}(w, Q^2) \hat{J}(v-w) dw, \quad (3.12)$$

Where  $\hat{J}(v) \equiv \mathcal{L}^{-1}[h(s)^{-1}, v]$ . The calculation of  $\hat{J}(v)$  by using the Eq. (3.7) and inverse Laplace transform of  $h(s)^{-1}$  for LO is straightforward and given in term of the Dirac delta function

$$\hat{J}(v) = 4 + 6\delta(v) + 2\delta'(v). \quad (3.13)$$

Using the Eq. (3.12) and Eq. (3.13) the polarized gluon distribution in v-space is given by

$$\hat{G}(v, Q^2) = 4 \int_0^v \hat{\mathcal{G}}(w, Q^2) dw + 6 \int_0^v \delta(v-w) \hat{\mathcal{G}}(w, Q^2) dw + 2 \int_0^v \delta'(v-w) \hat{\mathcal{G}}(w, Q^2) dw. \quad (3.14)$$

Using the following relation for Dirac delta function

$$\int_0^v \delta(v-w) f(w) dw = f(v), \quad (3.15)$$

$$\int_0^v \delta'(v-w) f(w) dw = f'(v). \quad (3.16)$$

we will have

$$\hat{G}(v, Q^2) = 4 \int_0^v \hat{\mathcal{G}}(w, Q^2) dw + 6\hat{\mathcal{G}}(v, Q^2) + 2 \frac{\partial \hat{\mathcal{G}}(v, Q^2)}{\partial v}. \quad (3.17)$$

Finally we have the polarized gluon distribution as

$$G(x, Q^2) = 4 \int_x^1 gg(z, Q^2) \frac{dz}{z} + 6gg(x, Q^2) - 2(x \frac{\partial gg(x, Q^2)}{\partial x}), \quad (3.18)$$

where  $gg(x, Q^2) = (\frac{\alpha_s}{2\pi} \sum e_i^2)^{-1} \mathcal{G}_1^p(x, Q^2)$  and  $\mathcal{G}_1^p(x, Q^2)$  is given by Eq. (2.2). We use 4 massless quarks (u,d,s,c) in our calculation and then  $\sum e_i^2 = \frac{10}{9}$ .

To calculate the right hand of the Eq. (3.18), we have to have an analytic function for  $g_1^p(x, Q^2)$ . So in the next section we try to do the global parametrization of  $g_1^p(x, Q^2)$  using all available experimental data and find this analytic function and then we calculate the polarized gluon distribution function numerically.

## 4. Numerical results

In this section we intend to use Eq. (3.18) to extract the polarized gluon distribution inside the proton.

### 4.1 Global parametrization of $g_1^p(x, Q^2)$ using all available experimental data

We have parameterized the polarized proton structure function,  $g_1^p(x, Q^2)$  in  $0 < x < 1$  as

$$xg_1^p(x, Q^2) = x^a(1-x)^b \frac{x_P g_P}{(1-x_P)^b x_P^a} (1 + A(Q^2) \text{Ln}[\frac{x_P(1-x)}{x(1-x_P)}]). \quad (4.1)$$

Here  $x_P = 0.234$  is an approximate fixed point observed in the data where the curves for different  $Q^2$  cross. At that point,  $\partial g_1^p(x_P, Q^2)/\partial \ln Q^2 \approx 0$  for all  $Q^2$ ;  $g_P = g_1^p(x_P, Q^2) = 0.241$  is the common value of  $g_1^p$ . We used all available experimental data for polarized proton structure function from E143, SMC, HERMES 2006 and COMPASS 2009 [18, 19, 20, 21, 22]. More accurate global fit with more data, over large kinematic ranges of  $x$  and  $Q^2$  will lead to the precisely determination of the polarized gluon distribution function. The  $Q^2$  dependence of  $g_1^p(x, Q^2)$  in our global fit is given by

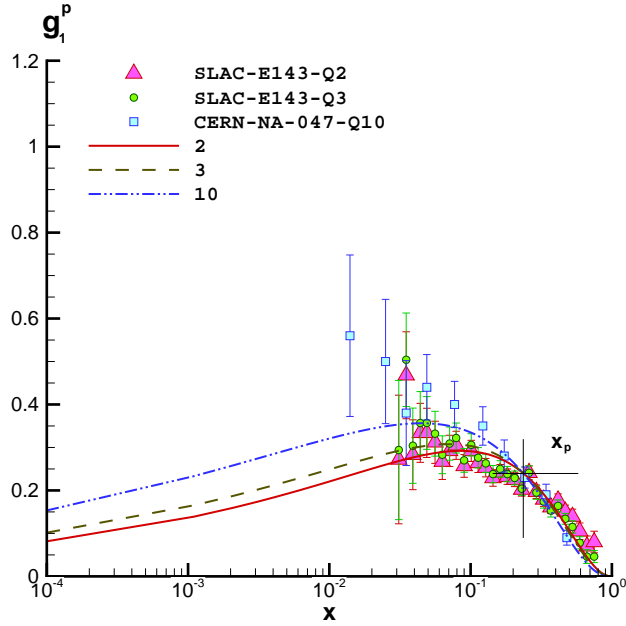
$$A(Q^2) = a_0 + a_1 \ln Q^2 + a_2 \ln^2 Q^2. \quad (4.2)$$

The fitted quantities are tabulated in Table 2.

We use the global fit in Eq. (4.1) and depict in Fig. 1 and Fig. 2 the polarized proton structure function for some values of  $Q^2$  and compared them with experimental data. The comparison indicates that the global fit works well.

<i>Parameters</i>	<i>valuse</i>
$a_0$	$-0.0287 \pm 0.1463$
$a_1$	$0.1253 \pm 0.0671$
$a_2$	$-0.0092 \pm 0.0145$
a	$1.2624 \pm 0.0979$
b	$2.227 \pm 0.4745$
$\chi^2(\text{Goodness of fit})$	0.984

**Table 2:** Global fit parameters obtained by fitting the Eq. (4.1) over the experimental data .



**Figure 1:** Polarized proton structure function,  $g_1^p$  for some values of  $Q^2$  extracted from global fit. Experimental data are from [18, 22]. The data cover the kinematic regions  $0.0041 < x < 0.9$  and  $0.18\text{GeV}^2 < Q^2 < 20\text{GeV}^2$ .

#### 4.2 Numerical results for the polarized gluon distribution

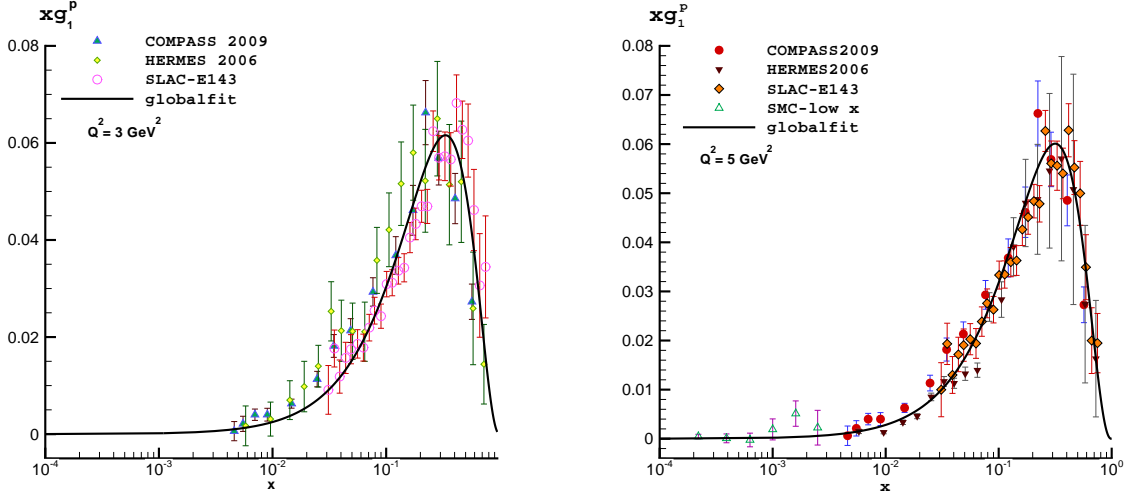
In this section we will briefly describe the numerical results for polarized gluon distribution by using the analytic solution, Eq. (3.18). Finally we will compare our result for  $x\delta g(x, Q^2)$  with those from *AAC'08*, *DSSV'08* and *LSS'06* global fits [23, 24, 25].

In this calculation, we use the LO approximation of  $\alpha_s(Q^2)$  which is defined in [17]

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}, \quad (4.3)$$

$$\beta_0 = 11 - \frac{2}{3}n_f, \quad (4.4)$$

with  $n_f = 5$  and  $\Lambda_5 = 146$  MeV for  $Q > 4.5$  GeV,  $n_f = 4$  and  $\Lambda_4 = 192$  MeV for



**Figure 2:** Polarized proton structure function,  $xg_1^p$  at  $Q^2 = 3\text{GeV}^2$  and  $Q^2 = 5\text{GeV}^2$  extracted from global fit. Experimental data are from [18, 19, 20, 21, 22, 27].

$1.3\text{ GeV} < Q \leq 4.5\text{ GeV}$ , and  $n_f = 3$  and  $\Lambda_3 = 221\text{ MeV}$  for  $Q < 1.3\text{ GeV}$ . These values have been used in *CTEQ5L* [26] and also used to extract the analytic unpolarized gluon distribution in [11].

Now by following these two steps we calculate the polarized gluon distribution:

- Calculating  $gg(x, Q^2)$  by using the Eq. (2.2):  $gg(x, Q^2) = (\frac{\alpha_s}{2\pi} \sum e_i^2)^{-1} \mathcal{G}_1^p(x, Q^2)$ . The convolution integrand in Eq. (2.2) with plus prescription,  $(\ )_+$ , can be easily calculated using [17]

$$\int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right)_+ g(y) = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) \left[g(y) - \frac{x}{y} g(x)\right] - g(x) \int_0^x dy f(y) \quad (4.5)$$

- By using the Eq. (3.18), we can extract numerically the polarized gluon distribution inside the proton .

Our result for polarized gluon distribution inside the proton is shown in Fig.3 (left) and compared with some global fits.

### 4.3 How to test the accuracy of the procedure ?

An independent method of checking the numerical accuracy of the entire procedure for extracting the polarized gluon distribution is to go back to the original DGLAP equation from which we started, Eq. (2.3), i.e.,

$$gg(x, Q^2) = x \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg}\left(\frac{x}{z}\right),$$

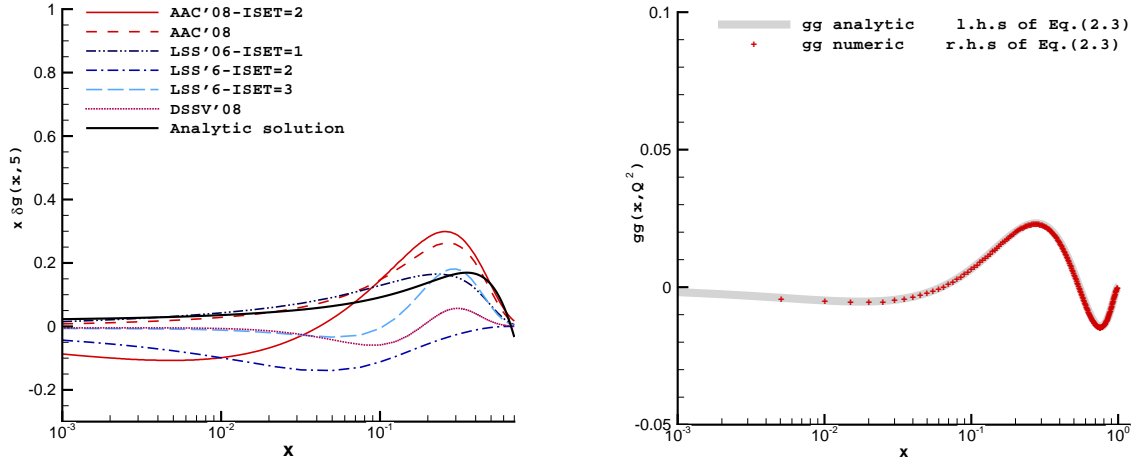


and numerically integrate its right hand side, which depends on our numerical solution . We then compare it with the  $gg(x, Q^2)$ , which is independently known and arising from our global fit for the polarized proton structure function,  $g_1^p(x, Q^2)$ , based on its relation to  $\mathcal{G}_1^p(x, Q^2)$  in Eq. (2.2). In Fig.3 (right) we plot the results of these different methods. Test of accuracy shows that our solution for polarized gluon distribution is correct (Eq. (3.18)). Of course, we should note that for using this solution for polarized gluon distribution, we need to do a global fit with experimental data for  $g_1^p(x, Q^2)$ . We do not have enough experimental data for small  $x$  ( $x < 10^{-2}$ ) also for large  $x$  ( $x > 0.7$ ), therefore we need to have  $g_1^p(x, Q^2)$  at these regions to be sure about the shape of polarized gluon distribution and future experiments should be focused on measuring the polarized structure function in these two regions.

The gluon contribution to the spin of the proton in our calculation is

$$\Delta g(Q^2 = 5\text{GeV}^2) = \int_{0.001}^1 \delta g(x, Q^2 = 5\text{GeV}^2) dx = 0.43 \quad (4.6)$$

This value indicates that the gluon contribution to the proton spin is considerable and compatible with other phenomenological models [23, 24, 25].



**Figure 3:** *Left:* Polarized gluon distribution function,  $x\delta g(x, Q^2)$  at  $Q^2 = 5\text{GeV}^2$  and comparison with other global fits. *Right:* Test of accuracy.

## 5. Conclusion

In this paper we tried to find an analytic solution for the DGLAP equation for polarized structure function and find the polarized gluon distribution inside the proton. Our solution is model independent and it is free of any parameters, initial input densities for solving the DGLAP equations and also free of the initial scale of  $Q_0^2$ . It is only dependent on finding an analytic function for  $g_1^p(x, Q^2)$  by using the experimental data. Our results suggest that the precise measurement of experimental values for  $g_1^p(x, Q^2)$  over large kinematic ranges of  $x$  and  $Q^2$  can predict directly the polarized gluon distribution more accurately.

## Acknowledgments

Authors are indebted to the institute for research in fundamental science (IPM) for their hospitality whilst this research was performed. We would like to thank G. Altarelli for his careful reading of the manuscript and for the productive discussions. We are grateful to M. Block for his useful suggestions, discussions and critical remarks. The authors are indebted to R. Sassot for giving us his useful and constructive comments.

## A. The DGLAP equation for the polarized proton structure function

In this part, we want to prove the Eq. (2.1).

$$x \frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2) \delta K_{qq} \left( \frac{x}{z} \right) = x \frac{\alpha_s}{2\pi} \sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg} \left( \frac{x}{z} \right), \quad (\text{A.1})$$

We begin with the DGLAP equations for the polarized parton distribution functions for quark and anti-quark sectors

$$\frac{\partial \delta q_i}{\partial t}(x, t) = \frac{\alpha_s}{2\pi} \left[ \int_x^1 \frac{dz}{z} \delta q_i(z, t) \delta K_{qq} \left( \frac{x}{z} \right) + \int_x^1 \frac{dz}{z} \delta g(z, Q^2) \delta K_{qg} \left( \frac{x}{z} \right) \right], \quad (\text{A.2})$$

$$\frac{\partial \delta \bar{q}_i}{\partial t}(x, t) = \frac{\alpha_s}{2\pi} \left[ \int_x^1 \frac{dz}{z} \delta \bar{q}_i(z, t) \delta K_{\bar{q}\bar{q}} \left( \frac{x}{z} \right) + \int_x^1 \frac{dz}{z} \delta g(z, Q^2) \delta K_{\bar{q}g} \left( \frac{x}{z} \right) \right], \quad (\text{A.3})$$

where  $t = LnQ^2$  and we used the following properties for the polarized splitting functions [17]:

$$\delta K_{qq} = \delta K_{\bar{q}\bar{q}}, \quad (\text{A.4})$$

$$\delta K_{qg} = \delta K_{\bar{q}g}, \quad (\text{A.5})$$

After summation of the Eq. (A.2) and Eq. (A.3) and multiply both side by  $\frac{1}{2} \sum_{i=1}^{n_f} e_i^2 x$ , we have

$$x \frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2) \delta K_{qq} \left( \frac{x}{z} \right) + x \frac{\alpha_s}{2\pi} \sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg} \left( \frac{x}{z} \right), \quad (\text{A.6})$$

In the Eq. (A.6) we have  $G(z, Q^2) = z \delta g(z, Q^2)$  and  $g_1^p(x, Q^2)$  is the LO polarized spin structure function for proton

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_{i=1}^{n_f} e_i^2 [\delta q_i(x, Q^2) + \delta \bar{q}_i(x, Q^2)]. \quad (\text{A.7})$$

Thus we achieved to our desired result in Eq. (2.1):

$$x \frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2) \delta K_{qq} \left( \frac{x}{z} \right) = x \frac{\alpha_s}{2\pi} \sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg} \left( \frac{x}{z} \right), \quad (\text{A.8})$$

## References

- [1] S. Procureur, Eur. Phys. J. A **32**, 483-487 (2007)
- [2] P. Liebing, "Can the Gluon Polarization in the Nucleon be Extracted from HERMES Data on Single High-pT Hadrons?", PhD thesis, Universit at Hamburg (2004).
- [3] HERMES Collaboration, A. Airapetian *et al.*, Phys. Rev. Lett. **84** (2000) 2584, hep-ex/9907020.
- [4] Spin Muon (SMC) Collaboration, B. Adeva *et al.*, Phys. Rev. D **70** (2004) 012002, hep-ex/0402010.
- [5] E.S. Ageev *et al.*, (COMPASS Collaboration) Phys. Lett. B **633** (2006) 25-32,
- [6] E.S. Ageev *et al.*, (COMPASS Collaboration), Nucl. Phys. B **765** (2007) 31.
- [7] S. Koblitz (2007), arXiv:0707.0175 [hep-ex]
- [8] Asymmetry Analysis Coll. (AAC), Y. Goto *et al.*, Phys. Rev. D **62**, (2000) 034017; *ibid* D **69**, (2004) 054021.
- [9] J. Blumlein and H. Bottcher, Nucl. Phys. B **636**, (2002)225.
- [10] M. Gluck, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D **63**, (2001) 094005.
- [11] M. M. Block, L.Durand, D.W. McKay, Phys. Rev. D **79**, 014031 (2009);
- [12] M. M. Block, L.Durand, D.W. McKay, Phys. Rev. D **77**, 094003 (2008)
- [13] M. M. Block, Eur. Phys. J. C **65** (2010): 17
- [14] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972).
- [15] G. Altarelli and G. Parisi, Nucl. Phys. B **126**, 298 (1977).
- [16] Y. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).
- [17] B. Lampe and E. Reya, Phys. Rep. **332** (2000) 1.
- [18] SMC Collaboration, B. Adeva *et al.*, Phys.Lett.B412(97)414
- [19] HERMES Collaboration, A. Airapetian *et al.*, Phys. Rev. D **75**: 012007(2007)
- [20] SMC Collaboration, B. Adeva *et al.*, Phys. Rev. D **58** 112001
- [21] SMC Collaboration, B. Adeva *et al.*, Phys. Rev. D **60**, 072004
- [22] E143 Collaboration, K. Abe *et al.*, Phys. Rev. L **75**, 1,(1995); Phys. Rev. D **58** (1998), 112003
- [23] M. Hirai, S. Kumano, Asymmetry Analysis Collaboration, Nucl. Phys. B **813**:106-122,2009
- [24] D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang, Phys. Rev. Lett. **101**:072001,2008.
- [25] E. Leader, A.V. Sidorov and D.B. Stamenov, Phys. Rev. D **75** (2007)
- [26] CTEQ Collaboration, H. L. Lai et al., Eur. Phys. J. C **12**, 375 (2000) [hep-ph/9903282].
- [27] COMPASS Collaboration, M. Alekseev *et al.*, hep-exp/1001.4654v1